Series : ABCD3/1

SET – 3

प्रश्न–पत्र कोड
Q. P. Code **65/1/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न–पत्र कोड को उत्तर–पुस्तिका के मुख–पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ **8** हैं ।
- प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न–पत्र में **14** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

- Please check that this question paper contains **8** printed pages.
- Q.P. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- **Please write down the Serial Number of the question in the answer-book before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

*

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

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सामान्य निर्देश :

निम्नलिखित निर्देशों को सावधानीपूर्वक पढ़ें और उनका अनुपालन करें ।

(i) *इस प्रश्न-पत्र के **तीन** खंड हैं – खंड **क, ख** तथा **ग** ।*

(ii) ***प्रत्येक** खंड **अनिवार्य** है ।*

(iii) ***खंड–क** में **6** लघु उत्तर–**I** प्रकार के प्रश्न हैं, जिनमें प्रत्येक के **2** अंक हैं ।*

(iv) ***खंड–ख** में **4** लघुउत्तर–**II** प्रकार के प्रश्न हैं, जिनमें प्रत्येक के **3** अंक हैं ।*

(v) ***खंड–ग** में **4** दीर्घ उत्तरीय प्रश्न हैं, जिनमें प्रत्येक के **4** अंक हैं ।*

(vi) *कुछ प्रश्नों में आंतरिक विकल्प दिए गये हैं ।*

(vii) *प्रश्न **14** एक प्रकरण अध्ययन प्रश्न है जिसमें दो भाग हैं, जिनमें प्रत्येक के **2** अंक हैं ।*

*

खंड – क

प्रश्न संख्या **1** से **6** तक प्रत्येक प्रश्न के **2** अंक हैं ।

1. माना A और B दो ऐसी घटनाएँ हैं जिनके लिए $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ तथा $P(A/B) = \frac{3}{4}$ है । P(B/A) का मान ज्ञात कीजिए । **2**

2. एक थैले, जिसमें 2 लाल गेंद और 3 नीली गेंदें हैं, में से यादृच्छया दो गेंदें बिना प्रतिस्थापना के निकाली जाती हैं । माना चर X लाल गेंदों की संख्या दर्शाता है । X का प्रायिकता बंटन ज्ञात कीजिए । **2**

3. अवकल समीकरण : $\left(x + \frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)^2 + 1$ की कोटि और घात का योगफल ज्ञात कीजिए । **2**

4. (a) यदि $\frac{d}{dx}[F(x)] = \frac{\sec^4 x}{\operatorname{cosec}^4 x}$ और $F\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$ हो, तो $F(x)$ ज्ञात कीजिए । **2**

अथवा

(b) ज्ञात कीजिए : $\int \frac{\log x - 3}{(\log x)^4}\, dx.$

5. λ के वे मान ज्ञात कीजिए जिनके लिए बिन्दु $(2, 1, \lambda)$ की समतल $3x + 5y + 4z = 11$ से दूरी $2\sqrt{2}$ इकाई है । **2**

6. यदि $\vec{a} = 2\hat{i} + y\hat{j} + \hat{k}$ और $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ दो ऐसे सदिश हो जिनके लिए सदिश $(\vec{a} + \vec{b})$, सदिश $(\vec{a} - \vec{b})$ के लम्बवत् है, तो y के सभी संभावित मानों को ज्ञात कीजिए । **2**

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General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *The question paper contains **three** Sections – Section **A**, **B** and **C**.*

(ii) ***Each** Section is **Compulsory**.*

(iii) ***Section – A** has **6** short answer type-**I** questions of **2** marks each.*

(iv) ***Section – B** has **4** short answer type-**II** questions of **3** marks each.*

(v) ***Section – C** has **4** long answer type questions of **4** marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question No. **14** is a case based problem with 2 subparts of **2** marks each.*

SECTION – A

Question numbers **1** to **6** carry **2** marks each.

1. Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A/B) = \frac{3}{4}$. Find the value of P(B/A). **2**

2. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability distribution of X. **2**

3. Find the sum of the order and the degree of the differential equation :

$$\left(x + \frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)^2 + 1$$ **2**

4. (a) If $\frac{d}{dx}[F(x)] = \frac{\sec^4 x}{\text{cosec}^4 x}$ and $F\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$, then find $F(x)$. **2**

OR

(b) Find : $\int \frac{\log x - 3}{(\log x)^4}\, dx$.

5. Find the values of λ, for which the distance of point $(2, 1, \lambda)$ from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units. **2**

6. If $\vec{a} = 2\hat{i} + y\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are two vectors for which the vector $(\vec{a} + \vec{b})$ is perpendicular to the vector $(\vec{a} - \vec{b})$, then find all the possible values of y. **2**

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खंड – ख

प्रश्न संख्या **7** से **10** तक प्रत्येक प्रश्न के **3** अंक हैं ।

7. (a) ज्ञात कीजिए : $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$. **3**

अथवा

(b) मान ज्ञात कीजिए : $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(4+\sin x)} dx$.

8. बिन्दुओं $\left(\frac{7}{2}, 0, 0\right)$, (0, 7, 0) और (0, 0, 7) से गुज़रने वाले समतल को रेखा $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ जिस बिन्दु पर काटती है, उस बिन्दु के निर्देशांक ज्ञात कीजिए । **3**

9. अवकल समीकरण : $\frac{dy}{dx} = \frac{x^2+y^2}{xy}$ को हल कीजिए । **3**

10. (a) यदि $\vec{a}$, $\vec{b}$, $\vec{c}$ और $\vec{d}$ ऐसे चार शून्येतर सदिश हैं जिनके लिए $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ और $\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d}$ है, तो दर्शाइए कि सदिश $(\vec{a} - 2\vec{d})$ और $(2\vec{b} - \vec{c})$ परस्पर समांतर हैं, जहाँ $\vec{a} \neq 2\vec{d}$, $\vec{c} \neq 2\vec{b}$ हैं । **3**

अथवा

(b) एक समांतर चतुर्भुज की दो संलग्न भुजाएँ $2\hat{i} - 4\hat{j} - 5\hat{k}$ एवं $2\hat{i} + 2\hat{j} + 3\hat{k}$ द्वारा निरूपित हैं । इस चतुर्भुज के विकर्णों के समांतर मात्रक सदिश ज्ञात कीजिए । विकर्ण-सदिशों का प्रयोग करके, समांतर चतुर्भुज का क्षेत्रफल भी ज्ञात कीजिए ।

खंड – ग

प्रश्न संख्या **11** से **14** तक प्रत्येक प्रश्न के **4** अंक हैं ।

11. ताश के 52 पत्तों की एक गड्डी से एक पत्ता कहीं खो जाता है । बाकी पत्तों से, बिना प्रतिस्थापना के दो पत्ते यादृच्छया निकाले जाते हैं और दोनों ही इक्के पाए जाते हैं । खो गये पत्ते के इक्का का पत्ता होने की प्रायिकता ज्ञात कीजिए । **4**

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SECTION – B

Question numbers **7** to **10** carry **3** marks each.

7. (a) Find : $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$. **3**

OR

(b) Evaluate : $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(4+\sin x)}\,dx$.

8. Find the co-ordinates of point where the line $\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}$ crosses the plane passing through points $\left(\frac{7}{2},0,0\right)$, (0, 7, 0) and (0, 0, 7). **3**

9. Solve the differential equation : $\frac{dy}{dx}=\frac{x^2+y^2}{xy}$. **3**

10. (a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are four non-zero vectors such that $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=4\vec{b}\times\vec{d}$, then show that $(\vec{a}-2\vec{d})$ is parallel to $(2\vec{b}-\vec{c})$ where $\vec{a}\neq 2\vec{d}$, $\vec{c}\neq 2\vec{b}$. **3**

OR

(b) The two adjacent sides of a parallelogram are represented by $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

SECTION – C

Question numbers **11** to **14** carry **4** marks each.

11. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace. **4**

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12. मान ज्ञात कीजिए : $\int_0^{\pi} \frac{x}{1+\sin x}\,dx.$ 4

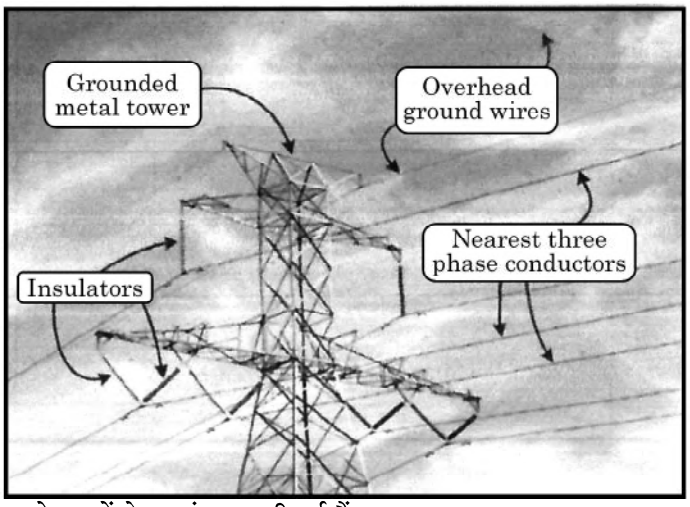
13. समाकलन के प्रयोग से, वक्र $y = x^2$, x-अक्ष तथा कोटियों $x = -2$ तथा $x = 1$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए। 4

अथवा

समाकलन के प्रयोग से, प्रथम चतुर्थांश में अर्ध–वृत्त $y = \sqrt{4-x^2}$, रेखा $y = \sqrt{3}x$ एवं x-अक्ष द्वारा घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

प्रकरण अध्ययन/आँकड़ा आधारित प्रश्न

14. विद्युत पारेषण तार जो सर्दियों में बिछाए जाते हैं, उन्हें गर्मियों में विस्तार को समायोजित करने के लिए कसकर फैलाया जाता है।



दो ऐसी तारे, निम्न रेखाओं के समांतर डाली गई हैं :

$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$

$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$

उपरोक्त सूचना पर आधारित होकर, निम्न प्रश्नों के उत्तर दीजिए :

(i) क्या रेखाएँ l_1 और l_2 सहतलीय हैं ? अपने उत्तर का औचित्य साबित कीजिए। 2

(ii) रेखाओं l_1 तथा l_2 का प्रतिच्छेदन बिंदु ज्ञात कीजिए। 2

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12. Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$. **4**

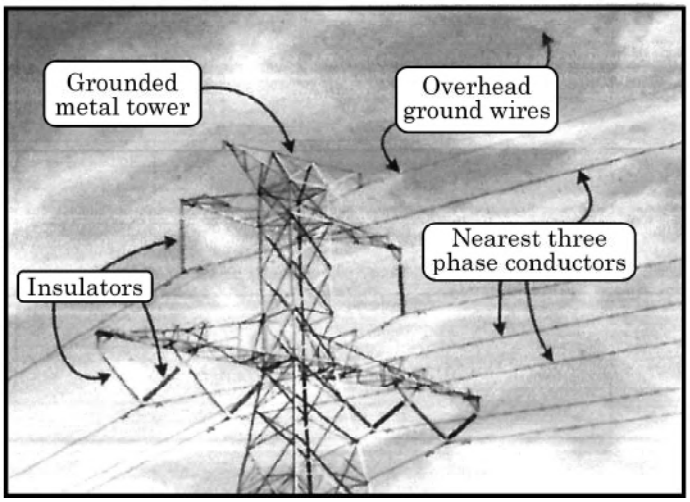
13. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$. **4**

OR

Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$, semi-circle $y = \sqrt{4-x^2}$ and x-axis in first quadrant.

CASE BASED / DATA BASED QUESTION

14. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.



Two such wires lie along the following lines :

$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$

$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$

Based on the given information, answer the following questions :

(i) Are the lines l_1 and l_2 coplanar ? Justify your answer. **2**

(ii) Find the point of intersection of the lines l_1 and l_2. **2**

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Senior Secondary School Term II Examination, 2022

Marking Scheme – MATHEMATICS (SUBJECT CODE – 041)

(PAPER CODE – 65/1/3)

General Instructions: -

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.

2. **"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under IPC."**

3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-XII, while evaluating two competency based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**

4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

5. Evaluators will mark (√) wherever answer is correct. For wrong answer 'X' be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**

6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

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10. A full scale of marks **0 TO 40** (example 0-40 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 30 answer books per day in main subjects and 35 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.

12. Ensure that you do not make the following common types of errors committed by the Examiner in the past: -
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.

14. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

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MARKING SCHEME

Senior Secondary School Examination TERM–II, 2022

MATHEMATICS (Subject Code–041)

[Paper Code: 65/1/3]

Maximum Marks: 40

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION – A	
1.	Let A and B be two events such that P(A) = $\frac{5}{8}$, P(B) = $\frac{1}{2}$ and P(A/B) = $\frac{3}{4}$. Find the value of P(B/A).	2
Sol.	$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{3}{4}\Rightarrow P(A\cap B)=\frac{3}{8}$	1
	$P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{\frac{3}{8}}{\frac{5}{8}}=\frac{3}{5}$	1
		2
2.	Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability distribution of X.	2
Sol.		

X	0	1	2
$P(X)$	$\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}$	$2\left(\frac{2}{5}\times\frac{3}{4}\right)=\frac{6}{10}$	$\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$

		1
		1
		2
3.	Find the sum of the order and the degree of the differential equation : $\left(x+\frac{dy}{dx}\right)^2=\left(\frac{dy}{dx}\right)^2+1$	2
Sol.	Given differential equation can be written as $x^2+\left(\frac{dy}{dx}\right)^2+2x\frac{dy}{dx}=\left(\frac{dy}{dx}\right)^2+1$	
	i.e. $x^2+2x\frac{dy}{dx}=1$; Order = 1, degree = 1	½ + ½
	Sum of order and degree = 1 + 1 = 2	1

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		2
4.	(a) If $\frac{d}{dx}[F(x)] = \frac{\sec^4 x}{\text{cosec}^4 x}$ and $F\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$, then find $F(x)$. **OR** (b) Find : $\int \frac{\log x - 3}{(\log x)^4}\, dx$.	2
Sol.	$F(x) = \int \tan^4 x\, dx = \int \tan^2 x.(\sec^2 x - 1)\, dx$ $= \int (\tan^2 x \sec^2 x - \sec^2 x + 1)\, dx$ $F(x) = \frac{\tan^3 x}{3} - \tan x + x + C$ $x = \frac{\pi}{4},\ F(x) = \frac{\pi}{4}$ gives $C = \frac{2}{3}$ $F(x) = \frac{\tan^3 x}{3} - \tan x + x + \frac{2}{3}$	½ 1 ½ 2
	Or Put $x = e^t$ or $\log x = t$ $\quad dx = e^t dt$ $\therefore \int \frac{\log x - 3}{(\log x)^4} dx = \int \frac{t-3}{t^4} e^t\, dt$ $= \int \left(\frac{1}{t^3} - \frac{3}{t^4}\right) e^t\, dt$ $= \frac{e^t}{t^3} + C$ $= \frac{x}{(\log x)^3} + C$	½ ½ ½ ½
5.	Find the values of λ, for which the distance of point $(2, 1, \lambda)$ from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units.	2
Sol.	$\frac{\lvert 3\cdot 2 + 5\cdot 1 + 4\lambda - 11 \rvert}{\sqrt{9+25+16}} = 2\sqrt{2}$ $\lvert 4\lambda \rvert = 20$ $\lambda = \pm 5$	1 1 2

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6.	If $\vec{a} = 2\hat{i} + y\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are two vectors for which the vector $(\vec{a} + \vec{b})$ is perpendicular to the vector $(\vec{a} - \vec{b})$, then find all the possible values of y.	2
Sol.	$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=0$ $\Rightarrow \lvert\vec{a}\rvert^2 - \lvert\vec{b}\rvert^2 = 0$ $\Rightarrow y^2+5-14=0$ $\Rightarrow y=+3 \text{ or } -3$	½ ½ ½ ½ 2
	SECTION – B	
7.	(a) Find : $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$. **OR** (b) Evaluate : $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(4+\sin x)}\,dx$.	3
Sol.	$I = \int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ $= \int \frac{6y^5 dy}{y^3+y^2}$ $\quad x = y^6$ so that $dx = 6y^5 dy$ $= 6\int \frac{y^3}{y+1} dy$ $= 6\int \left[(y^2-y+1) - \frac{1}{y+1}\right] dy$ $= 6\left[\frac{y^3}{3} - \frac{y^2}{2} + y - \log\lvert y+1\rvert\right] + c$ $= 2\sqrt{x} - 3\sqrt[3]{x} + 6x^{1/6} - 6\log(x^{1/6}+1) + c$	1 1 ½ ½ 3
	Or Let $\sin x = t$, then $\cos x dx = dt$ $\therefore \int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(4+\sin x)} dx = \int_0^1 \frac{dt}{(1+t)(4+t)}$	½ ½

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	$=\frac{1}{3}\left[\int_0^1 \frac{1}{1+t}dt-\int_0^1 \frac{1}{4+t}dt\right]$ $=\frac{1}{3}\left[\log(1+t)\Big\|_0^1-\log(4+t)\Big\|_0^1\right]$ $=\frac{1}{3}[\log 2-\log 5+\log 4]$ or $=\frac{1}{3}\log\frac{8}{5}$	1 ½ ½ 3
8.	Find the co-ordinates of point where the line $\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}$ crosses the plane passing through points $\left(\frac{7}{2},0,0\right)$, (0, 7, 0) and (0, 0, 7).	3
Sol.	Let the coordinates of required point on line is $(-\lambda+3,\ \lambda-4,\ 6\lambda-5)$ for some λ Equation of plane is $\frac{2x}{7}+\frac{y}{7}+\frac{z}{7}=1$ $\therefore\ 2(-\lambda+3)+\lambda-4+6\lambda-5=7$ Gives $5\lambda=10$ i.e. $\lambda=2$ $\therefore$ Required point is (1, –2, 7)	½ 1 1 ½ 3
9.	Solve the differential equation : $\frac{dy}{dx}=\frac{x^2+y^2}{xy}$.	3
Sol.	Let $y=vx$ $\quad\frac{dy}{dx}=v+x\frac{dv}{dx}$ $v+x\frac{dv}{dx}=\frac{1+v^2}{v}$ Gives $x\frac{dv}{dx}=\frac{1}{v}$ $v\,dv=\frac{dx}{x}$ Integrating we get $\frac{v^2}{2}=\log\|x\|+C$ $\frac{y^2}{2x^2}=\log\|x\|+C$	1 1 ½ ½

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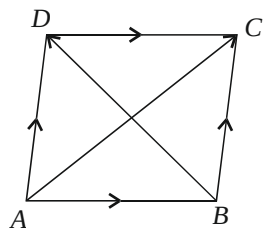
		3
10.	(a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are four non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d}$, then show that $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$ where $\vec{a} \neq 2\vec{d}$, $\vec{c} \neq 2\vec{b}$. **OR** (b) The two adjacent sides of a parallelogram are represented by $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.	3
Sol.	Consider $(\vec{a} - 2\vec{d}) \times (2\vec{b} - \vec{c})$ $= \vec{a} \times 2\vec{b} - \vec{a} \times \vec{c} - 4\vec{d} \times \vec{b} + 2\vec{d} \times \vec{c}$ $= 0$ $\therefore (\vec{a} - 2\vec{d}) \parallel (2\vec{b} - \vec{c})$	½ 1 1 ½ 3
	Or  Let ABCD be a parallelogram with $\overrightarrow{AB} = \overrightarrow{DC} = 2\hat{i} - 4\hat{j} - 5\hat{k}$ and $\overrightarrow{BC} = \overrightarrow{AD} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ and $\overrightarrow{BD} = 6\hat{j} + 8\hat{k}$ $\therefore \lvert\overrightarrow{AC}\rvert = 2\sqrt{6}$ and $\lvert\overrightarrow{BD}\rvert = 10$ $\therefore$ Required unit vectors d_1 and d_2 are $\hat{d}_1 = \frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{\hat{k}}{\sqrt{6}}$ and $\hat{d}_2 = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$ Now, Area of $\parallel ABCD = \frac{1}{2}\lvert\vec{d}_1 \times \vec{d}_2\rvert$ $= \frac{1}{2}\left\lvert\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & 6 & 8 \end{vmatrix}\right\rvert$	1 1

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	$= \frac{1}{2}\lvert -4\hat{i} - 32\hat{j} + 24\hat{k}\rvert$ $= \frac{1}{2}\sqrt{1616} = 2\sqrt{101}$	1 3
	SECTION – C	
11.	A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.	4
Sol.	E_1 : Lost card is ace E_2 : Lost card is not an ace A : 2 ace cards are drawn $P(E_1) = \frac{1}{13}$ $\quad P(E_2) = \frac{12}{13}$ $P(A/E_1) = \frac{{}^3C_2}{{}^{51}C_2}$ $\quad P(A/E_2) = \frac{{}^4C_2}{{}^{51}C_2}$ $P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$ $= \frac{\frac{1}{13}\frac{{}^3C_2}{{}^{51}C_2}}{\frac{1}{13}\cdot\frac{{}^3C_2}{{}^{51}C_2} + \frac{12}{13}\cdot\frac{{}^4C_2}{{}^{51}C_2}} = \frac{3}{75}$ or $\frac{1}{25}$	½ 1 1 1+ ½ 4
12.	Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x}\,dx.$	4
Sol.	$I = \int_0^{\pi}\frac{x}{1+\sin x}dx = \int_0^{\pi}\frac{\pi - x}{1+\sin(\pi - x)}dx = \pi\int_0^{\pi}\frac{1}{1+\sin x}dx - \int_0^{\pi}\frac{x}{1+\sin x}dx$ $\therefore\ 2I = \pi\int_0^{\pi}\frac{dx}{1+\sin x}$ $\therefore\ I = \frac{\pi}{2}\int_0^{\pi}\frac{dx}{1+\sin x} = \frac{\pi}{2}\int_0^{\pi}\frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)}dx$	1 ½ ½

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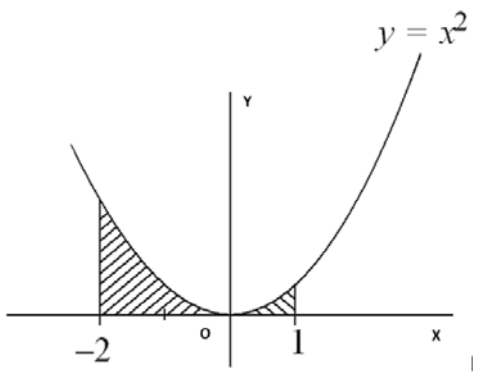
	$= \frac{\pi}{2}\int_0^{\pi} \frac{1}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}dx$ $= \frac{\pi}{4}\left[-2\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]_0^{\pi}$ $= \frac{\pi}{4}[2-(-2)] = \pi$	1 1 4
13.	Using integration, find the area of the region enclosed by the curve y = x^2, the x-axis and the ordinates $x = -2$ and $x = 1$. **OR** Using integration, find the area of the region enclosed by line y = $\sqrt{3}x$, semi-circle y = $\sqrt{4-x^2}$ and x-axis in first quadrant.	4
Sol.	 Correct figure $A = \int_{-2}^{1} x^2 dx$ $= \left.\frac{x^3}{3}\right]_{-2}^{1}$ $= \frac{1}{3} - \left(\frac{-8}{3}\right) = 3$	1 1 1 1 4
	Or	

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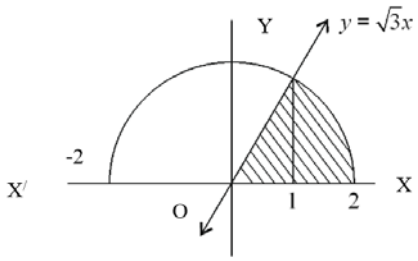


Correct figure 1

x coordinate of point of intersection is 1 ½

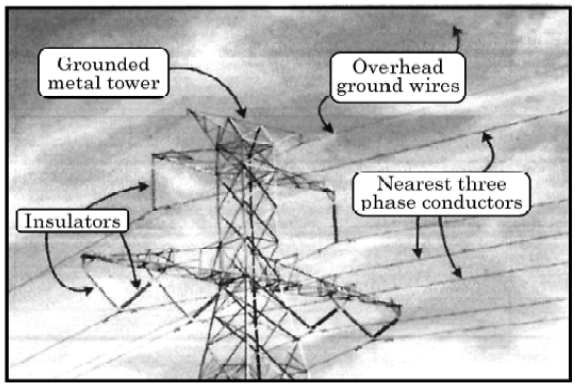
Required area $= \int_0^1 \sqrt{3}\, x\, dx + \int_1^2 \sqrt{4 - x^2}\, dx$ 1

$$= \sqrt{3}\left.\frac{x^2}{2}\right|_0^1 + \left.\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right|_1^2$$ 1

$$= \frac{\sqrt{3}}{2} + \pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$ ½

14. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.



Two such wires lie along the following lines :

$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$

$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$

Based on the given information, answer the following questions :

(i) Are the lines l_1 and l_2 coplanar ? Justify your answer. 2

(ii) Find the point of intersection of the lines l_1 and l_2. 2

Sol. *(i)* Consider $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$

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	$= \begin{vmatrix} +1 & 4 & -5 \\ 3 & -2 & -1 \\ -1 & 3 & -2 \end{vmatrix}$	1
	$= 1(+7) - 4(-7) - 5(7)$	
	$= 0$	
	$\therefore$ lines l_1 and l_2 are coplanar	1
	(ii) Any point on $l_1 : (3\lambda - 1, -2\lambda + 3, -\lambda - 2)$	1
	Substituting in equation of l_2,	
	$\frac{3\lambda - 1}{-1} = \frac{-2\lambda + 3 - 7}{3}$	
	$\Rightarrow 9\lambda - 3 = 2\lambda + 4$	
	$\Rightarrow 7\lambda = 7 \Rightarrow \lambda = 1$	½
	Point is $(2, 1, -3)$	½
		2 + 2

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